University of Tübingen

Physics Lab II

US

Ultrasound

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Contents

1 Abstract

In this experiment the generation and propagation of ultrasound in different materials is examined.

2 Theoretical Background

2.1 Elasticity

The behaviour of a material in which ultrasound propagates can be described using elastic coefficients, which are a direct measure of the resiliency of the material against deformation. Higher coefficients generally indicate stiffer materials.

2.1.1 Young's Modulus

Young's modulus E is defined in the context of an elastic body of length $l$as the ratio between the normal stress along the main axis $\sigma = F/A$ and the relative strain $\varepsilon = \Delta l/l$:

$$E = \frac{\sigma}{\varepsilon} = \frac{Fl}{A\Delta l} \tag{1}$$

2.1.2 Shear Modulus

The shear modulus G is defined in the context of an elastic body shearing by an angle γ as the ratio between shear stress $\tau = F/A$ and the tangens of the shear angle:

$$G = \frac{\tau}{\tan\gamma} = \frac{F}{\tan\gamma A} \approx \frac{F}{\gamma A} \tag{2}$$

2.1.3 Bulk Modulus

The bulk modulus K is defined in the context of an elastic body or fluid as the ratio between external pressure p and the relative change in volume:

$$K = -\frac{pV}{\Delta V} \tag{3}$$

(For positive pressures ΔV is negative and K positive!)

2.1.4 Poisson's Ratio

Poisson's Ratio ν is defined in the context of an elastic body of length l and diameter d as the ratio of relative change in diameter to reltaive change in length:

$$\nu = -\frac{\Delta d}{d}\frac{l}{\Delta l} \tag{4}$$

For materials that don't change diameter when stretched we have $\nu = 0$, dor incompressible materials $\nu = 0.5$.

The modules are related to each other according to the following formulae:

$$E = 2G(1+\nu) = 3K(1-2\nu) \tag{5}$$

2.2 Sound Waves

There are two basic types of (sound-) waves:
Longitudinal waves, where the particles of the medium oscillate along the propagation axis; and transverse waves, where they oscillate orthogonally.

In fluids no transverse waves can exist, because these media have no shear module since the molecules can't interact orthogonally. Longitudinal sound waves propagate with the following speed through fluids:

$$c_L^{\text{fluid}} = \sqrt{\frac{K}{\varrho}} \tag{6}$$

In solid bodies both kinds of sound waves can exist. They propagate at the following speeds:

$$c_L^{\text{solid}} = \sqrt{\frac{1-\nu}{1-\nu-2\nu^2}\frac{E}{\varrho}} \approx \sqrt{\frac{E}{\varrho}} \tag{7}$$

$$c_T^{\text{solid}} = \sqrt{\frac{G}{\varrho}} = \sqrt{\frac{1}{2(1+\nu)}\frac{E}{\varrho}} \tag{8}$$

We see that (usually) longitudinal waves will propagate faster than transverse waves!

2.3 Piezo Effect

Certain kinds of crystals will deform when put into a electric field. This can be used to create ultrasound, by simply putting such a crystal into a capacitor excited with an alternating current of frequency f. If this frequency hits a resonant frequency of the crystal, then the amplitude of the generated ultrasound will peak. Here we use a quartz-disk of length l where the k-th resonant wavelength λ_k corresponding to a standig wave with $k+1$ peaks is given by:

$$l = (2k+1)\frac{\lambda_k}{4} \qquad k \in N_0$$

Using that $\lambda_k = \frac{c_l}{f_k}$ and eq. (7), the resonant frequencies of the crystal are given by:

$$f_k = \frac{2k+1}{4l}\sqrt{\frac{E}{\rho}} \tag{9}$$

By submerging the excited quartz in ethanol we produce an ultrasound wave in the alcohol consisting of alternating areas with higher and lower pressure separated by distances equal to the wavelength λ. Because the refractive index n is a function of pressure, we have effectively created a refraction grating with $g = \lambda$, this is called the Debye-Sears-Effect.
Irradiating this grating with a laser of known wavelength λ_{Laser} produces an interference pattern on a screen placed on the opposite side. The maxima of intensity can be found at angles φ_k satisfying

$$\sin(\varphi_k) = \frac{k\lambda_{Laser}}{\lambda}$$

Let the distance between liquid and screen be d and the central distance of the k-th maximum x_k, then we have

$$\tan(\varphi_k) = \frac{x_k}{d}$$

In our experiment $x_k << d$, thus we get:

$$\lambda = \frac{kd\lambda_{Laser}}{x_k} \tag{10}$$

So by measuring the distances to the maxima we can calculate the wavelength of the ultrasound.

2.4 Magnetostriction

A ferromagnet is separated into areas of equal magnetic orientation, called the "magnetic domains" (German: "Weiss'sche Bezirke"). They have a purely statistical orientation, but when a magnetic field is applied, they align according to the direction of the field. Hence, the material experiences stress. This deformation can be measured. This effect is called magnetostriction.
If one places a rod-shaped ferromagnet, for instance nickel, into the center of a electromagnetic coil, one can measure the change in absolute length caused by the magnetostriction; this change can be calculated by:

$$\epsilon = \frac{\Delta l}{l} = \kappa\Gamma\frac{H}{E},$$

where $\kappa\Gamma$ is the magnetostriction constant and $H = \frac{NI}{L}$ is the magnetic field strength. Further we denote N as the number of loops and L as the length of the coil. After using these relationships and rearranging for $\frac{\Delta l}{I}$, one obtains

$$\frac{\Delta l}{I} = \kappa\Gamma\frac{Nl}{EL} \tag{11}$$

2.5 Longitudinal and transverse sound waves in Aluminium

In water, the longitudinal ultrasonic wave travels with velocity c_w and hits a cuboid made of aluminium.
If the wave's incident angle is rectangular, the wave traveling through the aluminium will also be a longitudinal wave. The instrument indicates a maximum followed by several small peaks caused by disturbance, for example the multiple reflection of the wave.
If, however, the ultrasonic wave hits under the angle $\alpha > 0$ with respect to the perpendicular plane, both a transversal and a longutidunal wave arise. Increasing the angle leads to a vanish of the longitudinal wave and solely the transversal wave is left. Following Snellius' law for two different velocities c_L, c_T and two different refraction angles β_L, β_T one obtains

$$\frac{\sin(\alpha)}{\sin(\beta_L)} = \frac{c_w}{c_L} \quad \text{und} \quad \frac{\sin(\alpha)}{\sin(\beta_T)} = \frac{c_w}{c_T}$$

The longitudinal wave vanishes at the angle $\beta_L = 90°$ due to total reflection. The transversal wave, on the other hand, peaks at $\beta_T = 45°$. The corresponding incident angles are denoted as α_T, α_L. The refractive law finally yields

$$c_L = \frac{c_w}{\sin(\alpha_L)} \quad \text{und} \quad c_T = \frac{c_w}{\sqrt{2}\sin(\alpha_T)} \tag{12}$$

The proportion of the longuitudinal wave can be expressed by the poisson number as follows:

$$x = \frac{c_L}{c_T} = \sqrt{\frac{2(1-\nu)}{1-2\nu}}$$

Rearranging for the poisson number yields

$$\nu = \frac{\frac{1}{2}x^2 - 1}{x^2 - 1}. \tag{13}$$

3 Experimental Procedure

The experiment consists of three parts:

1. First ultrasound standing waves are produced by a Piezo crystal in ethanol and made visible with a laser. Then the resonant frequenzies of the crystal and the material properties of the alcohol are determined, as well as the wavelength of the ultrasound.

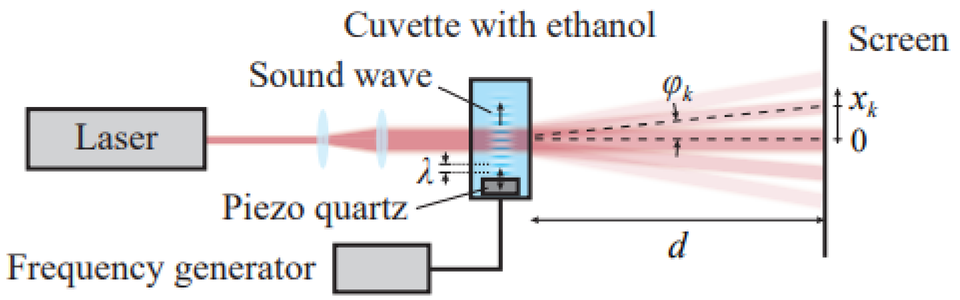

Figure 1: Setup part 1 (from the manual)

2. Now we consider magnetostriction. We fix a Ni-rod into a coil and alternate the current flowing through the coil to create magnetic fields that will change the length of the rod.

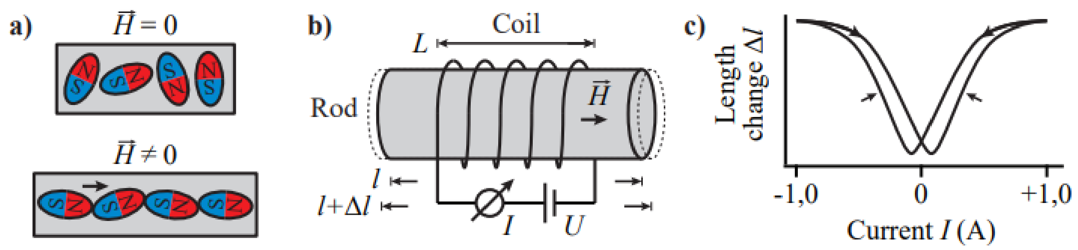

Figure 2: Setup part 2 (from the manual)

3. At last, we turn to the material properties of Al, like the sound speed. For this we simultaneously create longitudinal and transvere waves.

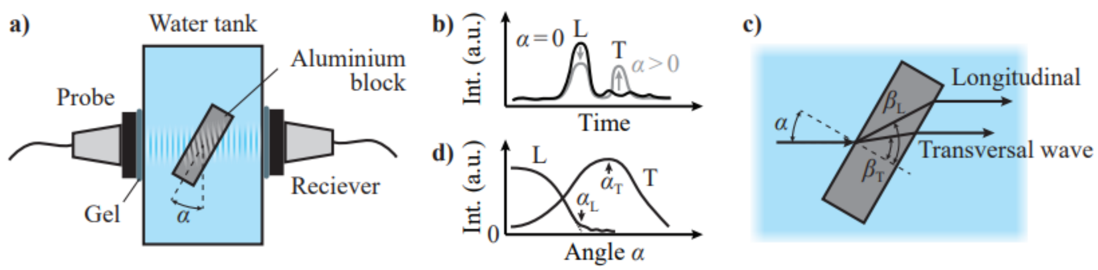

Figure 3: Setup part 3 (from the manual)

4 Results & Interpretaton

4.1 Piezo Effect

We found the following values for the first two resonant frequencies:

	$f_0[MHz]$	$f_1[MHz]$
mean	2.0622	6.0674
error	0.0283	0.0084

Table 1: Eigenfrequencies of the crystal

Re-arranging equation (9), we get the following two values for the length l of the crystal when using the two different frequencies:

	$l_0[mm]$	$l_1[mm]$
value	0.7065	0.7204
error	0.0097	0.0010

Table 2: Length of the crystal

So we get two slightly different results.

Additionally, we found the distance between two maxima for $f = f_1$ to be $\Delta x \approx (2.5 \pm 0.1)$mm. Together with the wavelength of the laser $\lambda_{Laser} = 632.8$nm and the distance fluid-screen $d \approx (70.0 \pm 1.0)$cm we can calculate the wavelength of the ultrasound at $f = f_1$, using eq. (10):

$$\lambda_1 = (177.2 \pm 9.6)\mu\text{m}$$
$$\Rightarrow c_{ethanol} = \lambda_1 \cdot f_1 = (1075 \pm 60)\frac{\text{m}}{\text{s}}$$
$$\Rightarrow K_{ethanol} = \varrho c_{ethanol}^2 = (0.912 \pm 0.10)\text{GPa}$$

The nominal value for the sound speed is $c_{ethanol} = 1207\frac{\text{m}}{\text{s}}$, which lies atop our value but well inside the error interval, for the modulus it's $K_{ethanol} = 0.9$GPa, which lies slightly below.

4.2 Magnetostriktion

It is possible to plot the change in length of the material with respect to the current I that runs through the coil:

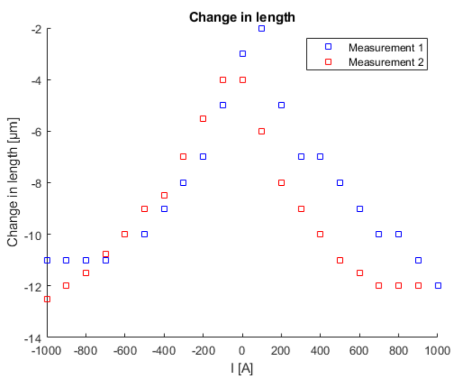

Figure 4: Change in length with respect to the current

Now we can fit it linearly four times, each measurement of the current two times respictively:

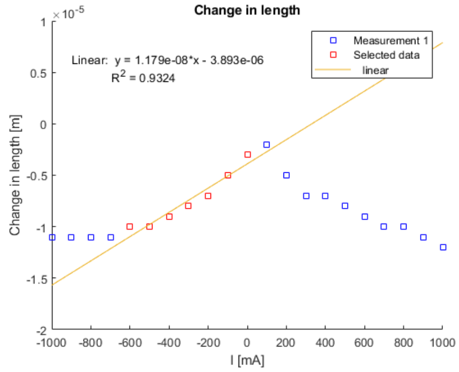

Figure 5: Change in length with respect to the current — including a linear fit

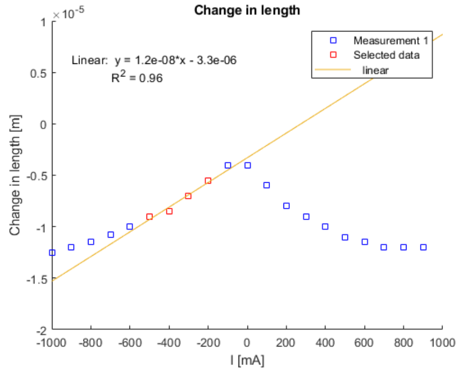

Figure 6: Change in length with respect to the current — including a linear fit

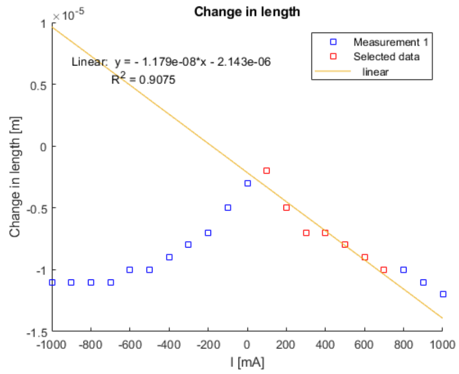

Figure 7: Change in length with respect to the current — including a linear fit

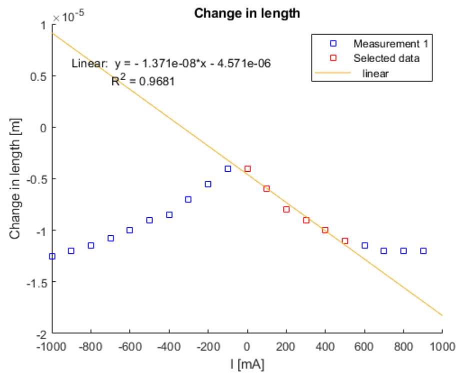

Figure 8: Change in length with respect to the current — including a linear fit

We can obtain the slope and the corresponding error to be:

Plot above	Slope m
1	$(1.179 \pm 0.08) \cdot 10^{-5} \frac{\mathrm{m}}{\mathrm{A}}$
2	$(1.2 \pm 0.048) \cdot 10^{-5} \frac{\mathrm{m}}{\mathrm{A}}$
3	$(-1.179 \pm 0.11) \cdot 10^{-5} \frac{\mathrm{m}}{\mathrm{A}}$
4	$(-1.371 \pm 0.44) \cdot 10^{-5} \frac{\mathrm{m}}{\mathrm{A}}$

Thus, by calculating the median of the absolute values we obtain

$$m = (1.23 \pm 0.5_{sys} + 0.17_{stat}), \tag{14}$$

where we estimated the systematic error as slightly larger than the largest statistical error. Using the relation

$$\kappa = m\frac{EL}{Nl}, \tag{15}$$

we arrive at

$$\kappa \approx (432.39 \pm 175.77_{sys} \pm 59.76_{stat}) \tag{16}$$

$$\frac{\Delta l}{l} \approx (27.95 \pm 1.14_{sys})\mu\mathrm{m}, \tag{17}$$

where we used the given values $E = 210\mathrm{GPa}$, $L = 39.7\mathrm{cm}$, $L = 5490$ and $l = 44\mathrm{cm}$. The value of κ is close to the value 500 from the literature and almost within the error interval of the statistical error. The systematic error was maybe estimated a bit too large. The relative change of length is much shorter than the expected value of around 40μm, which most likely results from some of the magnetic domains being arranged already before we started the experiment. We also did a test run, where we already aligned many of them.

4.3 Aluminium

The following figure shows a clear pattern that indicates the maximum of the transversal wave at low angles α, while the amplitude for longitudinal waves increases for large α where the transversal one decreases.

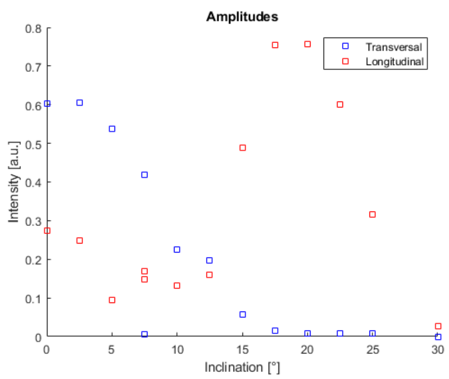

Figure 9: Intensity of the longitudinal / transversal wave with respect to the inclination angle

The angles α_T and α_L, where the transversal wave and the longitudinal wave vanish and peak respictively can visually be extracted from the figure above. Hence, one finds

$$\alpha_T \approx 17.5 \pm 1.25 \tag{18}$$

$$\alpha_L \approx 17.5 \pm 1.25, \tag{19}$$

where the error has been estimated by choosing it as 25% of the smallest scale one can identify, i.e. steps of 5. Now we can calculate the sound speeds, the Young module $E \approx \rho c_L^2$, the torsion module $G = \rho c_T^2$ and the poisson number ν to be

$$\begin{aligned}
c_L &= \frac{c_W}{\sin(\alpha_L)} \pm \Delta c_L \approx (4921.75 \pm 340.55)\frac{\mathrm{m}}{\mathrm{s}} && (20)\\
c_T &= \frac{c_W}{\sqrt{2}\sin(\alpha_T)} \pm \Delta c_W \approx (3480.21 \pm 240.81)\frac{\mathrm{m}}{\mathrm{s}} && (21)\\
E &\approx \rho c_L^2 \approx (65.40 \pm 6.40)\mathrm{GPa} && (22)\\
G &\approx \rho c_T^2 \approx (32.70 \pm 6.40)\mathrm{GPa} && (23)\\
\nu &\approx (0 \pm 0.294), && (24)
\end{aligned}$$

where we used that $\rho \approx 2.7 \cdot 10^{-6} \frac{\mathrm{kg}}{\mathrm{m}^3}$ and the sonic speed in water $c_W \approx 1480\frac{\mathrm{m}}{\mathrm{s}}$. The errors were calculated by using the propagation of uncertainty. All in all they fit very well to the values from the literature, except ν, which could arise from the large error interval, most likely resulting from the uncertainties when adjusting the angles on a rough scale.